Theorem 1 (Robert Gerbicz) *Let $q > 3$ prime and $p = W(q) = \frac{2^q+1}{3}$ is also prime (Wagstaff prime), then for the sequenc $S_0 = \frac{3}{2}$, $S_{k+1} = S_k^2 - 2$ it is true that $S_q - S_1$ is divisible by p.*

It is the same as the original conjecture for $U_0 = \frac{1}{4}$, $U_{k+1} = U_k^2 - 2$, because in our sequence $S_1 = U_0$ and the recursion is the same, so $U_{q-1} - U_0 = S_q - S_1$.

It is known that $T = Q[\sqrt{-7}] = Q[\sqrt{7} * I]$ is a prime factorization field. First I prove:
Lemma: Suppose that p is an odd positive prime in Z, different from 7, a and b are integers in Z, let $z = \frac{a+b\sqrt{7}*I}{4}$ then if $(\frac{-7}{p}) = 1$ and $gcd(z, p) = 1$ then $z^{p-1} \equiv 1$ mod p. If $(\frac{-7}{p}) = -1$ then $z^{p+1} \equiv norm(z)$ mod p.

Proof: use Fermat's little theorem and that binomial(p,k) is divisible by p if $0 < k < p$.
$(4z)^p \equiv 4^p z^p \equiv 4z^p$ mod p.
$(4z)^p \equiv (a+b\sqrt{7}I)^p \equiv a^p + b^p \sqrt{7}^p I^p \equiv a + b7^{\frac{p-1}{2}} \sqrt{7}(-1)^{\frac{p-1}{2}} I \equiv a + b(\frac{-7}{p})\sqrt{7}I$ mod p
so $4z^p \equiv 4z$ or $4\overline{z}$ mod p.
First part: let $(\frac{-7}{p}) = 1$ then we can write: $4z^p \equiv 4z$ mod p if $gcd(z, p) = 1$ then $z^{p-1} \equiv 1$ mod p.
Second part: let $(\frac{-7}{p}) = -1$ then $4z^p \equiv 4\overline{z}$ mod p, multiple this by z, we get: $4z^{p+1} \equiv 4norm(z)$ mod p, so $z^{p+1} \equiv norm(z)$ mod p is also true.
Proof of the lemma is complete.

Proof of the theorem 1: Let $\omega = \frac{3+\sqrt{7}I}{4}$ an element in the field. Let $S_0 = \frac{3}{2}$ and $S_{k+1} = S_k^2 - 2$, by induction it is easy to see, that $S_k = \omega^{2^k} + \overline{\omega}^{2^k}$, for this use that $norm(\omega) = 1$
If $q > 3$ prime then $q = 6k + -1$, so $p = W(q) = \frac{2^q+1}{3} \equiv 11$ or 15 mod 28, using this we can get that $(\frac{-7}{p}) = 1$. Use the lemma for $z = \omega$ and for $p = W(q)$ prime, we obtain:
$\omega^{W(q)-1} \equiv 1$ mod p
$\omega^{\frac{2^q-2}{3}} \equiv 1$ mod p raise it to cube:
$\omega^{2^q-2} \equiv 1$ mod p multiple it by ω^2
$\omega^{2^q} \equiv \omega^2$ mod p, conjugate it:
$\overline{\omega}^{2^q} \equiv \overline{\omega}^2$ mod p Adding these two lines: $S_q = \omega^{2^q} + \overline{\omega}^{2^q} \equiv \omega^2 + \overline{\omega}^2 = S_1$ mod p, so $S_q - S_1$ is divisible by p, proof is complete.

Theorem 2 (Robert Gerbicz) *Let $S_0 = -\frac{3}{2}$ and $S_{k+1} = S_k^2 - 2$ sequence. If $p = F(n) = 2^{2^n} + 1$ is a Fermat prime then $S_{2^n} - S_1$ is divisible by p.*

This is almost the original conjecture (that was: $S_{2^n-1} - S_0$ is divisible by p).

Proof of the theorem 2:

For $n = 0$ it is true. Now suppose that $n > 0$ and replace S_0 by $-S_0$, and by this we get the same sequence for S_k, if $k > 0$. But this sequence is the same as the S sequence was for the Wagstaff primes. $p = F_n \equiv 5$ or 17 mod 28, using this it is easy to see, that $(\frac{-7}{p}) = -1$, $norm(omega) = 1$, $gcd(norm(omega), p) = gcd(1, p) = 1$, using the lemma:

$\omega^{p+1} \equiv 1$ mod p

$\omega^{2^{2^n}+2} \equiv 1$ mod p, multiple it by $\overline{\omega}^2$

$\omega^{2^{2^n}} \equiv \overline{\omega}^2$ mod p, conjugate it:

$\overline{\omega}^{2^{2^n}} \equiv \omega^2$ mod p

Add the two lines: $S_{2^n} = \omega^{2^{2^n}} + \overline{\omega}^{2^{2^n}} \equiv \overline{\omega}^2 + \omega^2 = S_1$ mod p. So $S_{2^n} - S_1$ is divisible by p. The proof is complete.